Mathematics Education: Is it ethical?

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In February/March 2012 I spent a month in South Africa at the invitation of a South African University. I spent the time delivering a series of seminars and working with university staff and doctoral and masters students, mainly on issues of theory and methodology. At one point staff members asked me for ideas on creating access programmes for African South African students, who were overwhelmingly overrepresented among those failing their courses in mathematics education. What kind of advance preparation would these students need if they were to become better prepared? My reply was to wonder whether they (the staff) were asking the right question. Had it occurred to them to reflect on what it might be in the curriculum, including their teaching and, particularly their assessment practices that might be responsible for the students' failures? Apparently, it had not. This was not an unexpected response. The curriculum at any level of schooling—including Higher Education—is generally a given and stands as that against which students are to be measured. If this measurement registers a failure, then there must be a deficit in terms of student readiness or ability. Now, if their courses had been in humanities or arts subject, then my question might have been taken more seriously. After all, the content of a history curriculum, for example, might have been expected to encounter political challenges, but mathematics seems widely to be taken as perhaps difficult, but generally non-contentious and necessary and mathematics education surely concerns the transmission and acquisition of its essential skills. So, who is going to argue with that? Well, this is a rather simplistic view of pedagogy: here is Pádraig Hogan in a more reflexive mood:

> … as a teacher I might quite rightly decry the determined resistance of my students as obstructions of my efforts to teach them maths. But I might also fail to notice that my own understanding of mathematics – as essentially a matter of mastering procedures and rules – might be a crucial contributory factor. Allied to this might be a failure on my part to appreciate the importance of involving my students in active ways in their own learning. I might also fail to see that this would mean cultivating practices of learning that embody practical forms of justice, and the progressive sharing by my students of more of the responsibility for their learning. Even if I do appreciate this, I might not see the further practical consequences. I might not realise that teaching and learning, far from being essentially a matter of transmission and reception, constitute a joint event, experienced from a range of different perspectives by those involved. I might not notice a necessity to develop new kinds of learning relationships that disclose more imaginatively the topic being studied and that combine this with manifestations of fair play in the learning experiences themselves.
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> (Hogan, 2010, pp. 90-91)

Hogan is questioning the traditional teacher-student relationship and the transmission/reception interpretation of education and suggesting that this might, at least in part, be responsible for student resistance to reception. 'Reception' seems, like transmission, to be associated with messaging by radio or other media. Acquisition, on the other hand is, after all is what learning is and I prefer this term to the dominant emic word, 'learning', provisional on its decoupling from 'transmission'. It is 'transmission', not 'acquisition', that locates control with the teacher and denies it to the student.

Hogan's challenge is, of course, consistent with the arguments of liberal educators, including Dewey, Piaget and Schön and popular in contemporary, Western educational theory, but not universally so. Kanako Kusanagi (2022) presents evidence to this effect in her book, in which she also presents her ethnographically inspired research on the import of the Japanese teacher development programme, 'lesson study' to Java, Indonesia. 'Lesson Study' in its current form is fundamentally liberal in its design, but this ran counter to the social relations and cultural practices that prevailed in the Javanese school in which Kusanagi's study was based. The result of this antagonism was that, whilst the sternly bureaucratic organisation of the school ensured that the special lesson study lessons were organised as dictated by the programme, this had no effect on the everyday lessons in the school, so there was no impact on teacher development. It might be claimed that a liberal programme had been introduced in a decidedly anti-liberal way.

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The social relations in Kusanagi's study school were constituted, officially, by a rigid hierarchy maintained by bureaucratically distributed responsibilities. Those responsibilities that were not simply administrative involved presenting the curriculum to the students—transmission. Acquisition was essentially the students' responsibility. The teacher's official identity was as a civil servant and not a pedagogue. Relations outside of the official hierarchy were flatter. Kusanagi describes a familial culture in which individuals bore responsibility to support the teacher collective, though not to support the students, which, in effect, was optional. This division of labour: teachers/ transmission versus students/acquisition combined with an official hierarchical bureaucracy and an unofficial, flat familial structure is clearly an inhospitable environment for addressing ethical concerns regarding the curriculum insofar as these concerns relate to the relations between teachers and students, pending fundamental changes in the sociocultural regulation of schooling in Java. There are, however, features of schooling in Indonesia and more widely that are not under the direct control of the teachers. Most obviously, there is the matter of educational materials, including textbooks, that are produced outside of the school.

Mathematics textbooks come under ethical scrutiny when it is found that they represent socially different categories of students differently or when, for example, they code mathematical ability in terms of social distinctions. I (for example, Dowling, 1998) identified both of these features in the school mathematics scheme (*SMP 11-16*[1]) that was widely used in England and Wales in the 1980s and early 1990s. In Dowling & Burke (2012) we introduced the scheme in Figure 1. School students fall naturally into social categories, including—socioeconomic status (ses); gender; race; (dis)ability. These categories are sometimes (not always) visually apparent and where a category is visually apparent, the respective images transmit the signified category either by its presence or its absence so that students are able to see how their category is represented or that it is absent. This is an ethical issue. For example, femininity was notable by its *invisibility* in school mathematics texts in the UK until comparatively recently and represented in *stereotype,* even in the 1980s when there was also evidence of *tokenism*. The same was true of racial diversity and diversity in terms of ablebodiedness.

[1] Published by Cambridge University Press.

A more subtle form of differentiation is apparent on deploying the relational space in Figure 2. I refer to my analysis of the school mathematics scheme, *SMP 11-16*. From the third year of secondary schooling, the scheme is organised in terms of 'ability'. My analysis demonstrated that the scheme recognised 'ability' in terms of ses. This was apparent in the images and settings that were represented in each of the 'ability' tracks. The four categories in Figure 2 can be described as follows. The *esoteric domain* consists of text that is strongly institutionalised in terms of expression (ie mathematical symbols, terms and diagrams) and content (mathematical concepts). It is only in this domain that mathematical principles and rules can be fully realised. In contrast, the *public domain* consists of non-mathematical expression and content and so includes everyday settings, such as shopping and other settings drawn from a diverse range of non-mathematical contexts. These settings, however, are organised according to *esoteric domain* principles, so that, for example, shopping in these texts is not shopping as practiced by shoppers (see Lave *et al*, 1984), but recontextualised shopping; it is, in effect, a fictionalised shopping. The other two domains are hybrids. The *descriptive domain* employs mathematical expressions to signify non-mathematical objects, relations and practices—this is the domain of mathematical modelling. The *expressive domain* signifies mathematical objects and relations using non-mathematical signfiers—this is the domain of metaphors such as a fraction is a piece of cake or an equation is a balance; metaphoric signifiers always detach from their signifieds if pushed too far, which, in the case of *expressive domain* metaphors leads to confusion rather than enlightenment (Dowling. 2007).

Figure 1. Strategies of Representation (From Dowling & Burke, 2012)

	Orientation to Pattern	
Expression	consonance	dissonance
Connotative (tacit)	*invisibility*	*tokenism*
Denotative (explicit)	*stereotype*	*interrogation*

Figure 2. Domains of Action (from Dowling 2009)

	Content	
Expression	I^+	I^-
I^+	*esoteric domain*	*descriptive domain*
I^-	*expressive domain*	*public domain*

I stands for strength of institutionalisation.

Analysing the tracked books from *SMP 11-16*, I found that the lowest track text—designed for 'low ability'/low ses students—remained almost entirely in the *public* *domain*. The highest track text—designed for 'high ability'/high ses students—by contrast, moved between the *public* and *esoteric domains* with substantial textual time spent in the *esoteric*. This structure became progressively more pronounced in the moves between the first and last books in each track. The effect of these trajectories was to provide 'high ability'/ses students a career into *esoteric* mathematics and to trap the 'low ability'/ses students in fictional, quotidian discourse. The mathematics education of the day translated socioeconomic status into ability. Was this ethical?

But there is another twist! The *esoteric domain* of school mathematics is essentially a self-referential collection that has no substantial references either horizontally, to other school subjects (which, if they recruit mathematical expression/content always recontextualise it), or vertically, to mathematics in Higher Education (which departs almost entirely from school mathematics). Thus, both low ses and high ses students are inducted into fictional discourses, though the latter do have the advantage of the acquisition of symbolic capital (Bourdieu, 2021) that might get them into university, where, basically, they'll have to start over!

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This is how the purpose of studying mathematics at school (KS3 & 4, for ages 12-16) is presented in the National Curriculum:

> Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.[2]

Taken as a whole, this statement exhibits what I (Dowling, 1998) have referred to as the 'myth of participation', claiming that mathematics is essential, critical, necessary for participation in diverse activities that, together range over most areas of our lives: without mathematics, it seems, we are incapable of living adequately. Well, it is true that almost everything can be described in mathematical terms, but that is not the same as saying that we need school mathematics in order to participate in these activities. I have referred to the publication by Jean Lave *et al* that provides examples of everyday activities that are performed imaginatively, creatively, but that seem to owe nothing to school mathematics. Some advocates of so-called 'ethnomathematics' describe in mathematical terms practices in a range of settings in non-industrial societies demonstrating, they claim, that these societies have actually 'discovered' mathematics (see Dowling, 1998). Well, an article in the *Observer* newspaper recently[3] reported that bees (the insects) are capable of counting; as far as I am aware, no bee has ever passed a school mathematics test. Not only that, but the growth of the celery plant follows the Fibonacci sequence without the benefit of a mathematics education. These are flippant examples, of course, but in her study of Atlantic African cultures, in

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[2] https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/239058/SECONDARY_national_curriculum_-_Mathematics.pdf

[3] https://www.theguardian.com/environment/2022/jul/16/bees-are-really-highly-intelligent-the-insect-iq-tests-causing-a-buzz-among-scientists

which qualitative value is not always automatically realisable numerically, the anthropologist, Jane Guyer (2004) presents evidence of societies in which the ‘calculation’ of value must recognise culturally specific obligations and rights that from a Western point of view, but not from a local perspective, are likely be (mis)interpreted as corruption. The hegemony of the mathematical computation of value is not universal, though Western political domination pretty much is. Guyer concludes that:

> Scalar representations of judgmental concepts—of the goods at issue (quality), of the services they bring (utility, benefit) and the costs their acquisition incurs (risk)—have to be reformulated to be expressed numerically. Some reductions of scalar value to number are historical events, the results of political proclamation. Others are continuously institutionalized by dramatic performances, such as competitions, which in turn ripple out their effects into new scalar formations: for rules of qualification as well as excellence, for judges, for arenas, for training institutions, for materials, for expertise in the materials, and so on. Formal competitions are relentless generators of new scales of value by drawing participants and audience into a scalar logic, a monetary logic, and a mode of binding the two. Gradations are assigned a numerical referent, which affects prize money, stud fees, consultancy fees, access to other resources such as book contracts and foreign markets, and so on. Even when the numerical translation of the ordinal scale is implausible—beauty from 1 to 10? two 5's of beauty as good as one10?—the capacity of numbers to express other values is now a hegemonic idea in the modern economy, enforced by law and inculcated by competitions and professional organizations. After initial struggles over the terms for each new domain (orchids, aerobics, for example), the equation of qualitative and monetary scales eventually erases the constructions and disjunctures that have been overridden. It is only by a massive discounting of the "tournament of value" that we can retain the notion of the theoretical dominance of supply and demand in "markets" as the main representation of the operations of value in modern economies.
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> (Guyer, 2004; p. 52)

Is this a presentation of ‘financial literacy’ and so the ‘necessary’ mathematics education that is to provide access to it as ethical practices? Well, I can hear the counter to this, “This is how the world works here and now and we have a duty to prepare our children to participate in it.” Do we? Presumably, then, it’s OK for the Japanese Yakuza to apprentice their novices into their own particular forms of violence.

The alternative, of course, is to introduce a critical dimension to our mathematics education and there are many well-meaning mathematics educators who take precisely this line. Unfortunately, this is more easily attempted than achieved as illustrated by our (Dowling & Burke, 2012) discussion above of the relational space in Figure 1 illustrates. In Dowling (2010) I include a brief discussion of an account of a lesson by Eric Gutstein (2002). The lesson involved the use of graphing calculators to explore the statistical concept, ‘expected value’ by looking at data on police traffic stops, classified by the race of the driver, in the US State of Illinois. The students concluded that, if the stops were random, then the number of stops of Latino drivers should be consistent with the proportion of drivers who were Latinos. They were not: Latino drivers were substantially over-represented by a factor of about 4. Does this demonstrate that the Illinois Police were acting in a racist way? Well, its’s not altogether clear, because random traffic stops are illegal in the US, being a breach of Fourth Amendment rights that maintain that the police must be able to demonstrate ‘probable cause’ of an offence having been committed. So, the traffic stops should not have been random. It may have been, however, that a correlation between race and relative poverty meant that Latinos were more likely than Caucasians to be driving elderly and poorly maintained vehicles having visible defects, or even defective speedometers, and that this was responsible for the excess traffic stops.

Interestingly, one Illinois Police Department used a similar analysis of their traffic stops, by ethnicity and gender to argue that the stops of different categories were in proportion to their respective representation in the community and so:

> Wilmette police officers [were] engaging in bias free traffic enforcement'
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> Carpenter, 2004. P. 66)

The stops by the Wilmette Police were presumably, also not random/illegal, which raises the question of possible quota traffic stops!

The problem with Gutstein's lesson was that it was a mathematics lesson, prioritising a mathematics curriculum theme, and not an exploration of social justice. Had it been the latter, then the mathematical modelling should also have come under scrutiny, not just, what are random numbers, but is their use to generate metrics appropriate in this context? This illustrates the general difficulty with *public domain* teaching: it's not about 'the real world', it's about the mathematised world. Whether the setting is shopping or social injustice, is the parading of the mathematised world in the guise of the real world ethical?

This does not happen only in mathematics education. Health is a regular theme in the news media. Here's a headline from The *Guardian*: 'Extra glass of wine a day 'will shorten your life by 30 minutes' (The Guardian. 2018.04.13) such claims are ludicrous, untested and indeed untestable as the lives used to make such a claim must of necessity already have ended and 'your life' has not yet ended and so its length is unknown; it's a prediction that is essentially probabilistic and grounded on retrospective data. Probability is calculated on the basis of equally likely events, so each face of a cubic die is equally likely to land face-up when the die is tossed: a probability of 1/6. Now, human response to alcohol is a seriously complex issue and the specific outcomes of alcohol consumption by 6 (or even 600) individuals are likely all to be different. This study may be informative:

> Alcohol consumption level and alcohol use disorder (AUD) diagnosis are moderately heritable traits. We conduct genome-wide association studies of these traits using longitudinal Alcohol Use Disorder Identification Test-Consumption (AUDIT-C) scores and AUD diagnoses in a multi-ancestry Million Veteran Program sample (N = 274,424). We identify 18 genome-wide significant loci: 5 associated with both traits, 8 associated with AUDIT-C only, and 5 associated with AUD diagnosis only. Polygenic Risk Scores (PRS) for both traits are associated with alcohol-related disorders in two genetic correlation reflects the overlap between the traits, genetic correlations for 188 non-alcohol-related traits differ significantly for the two traits, as do the phenotypes associated with the traits' PRS. Cell type group partitioning heritability enrichment analyses also differentiate the two traits. We conclude that, although heavy drinking is a key risk factor for AUD, it is not a sufficient cause of the disorder.
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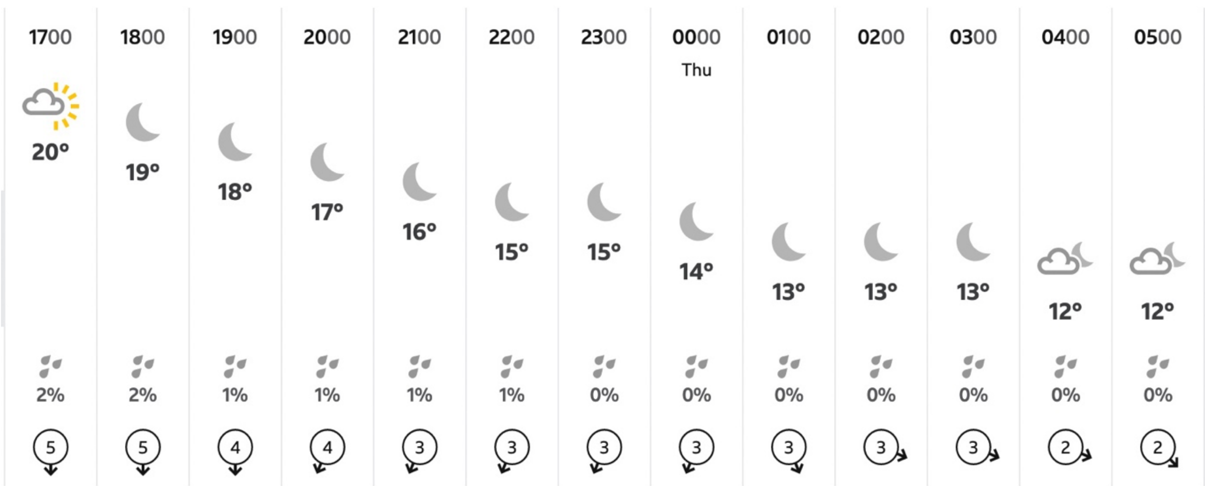
> Kranzler, H.R. et al. 2019

The *esoteric domain* language and, in general, the argument of this article will exclude many, if not most readers, who will need to rely on the headline; certainly, a school mathematics education won't help them. How should they respond? When researchers speak with each other in the same discipline, they speak in their specialist, *esoteric domain* discourse. When they speak in public, they of necessity speak in *public domain* discourse. School mathematics won't be of much use, because, when it addresses non-mathematical settings, it also speaks in the *public domain.* Is this democratic, is it ethical?

There is a particular issue with the teaching of probability in school mathematics that is discussed in Dowling, 1995[4]. This chapter presents an analysis of those chapters of *SMP 11-16* that deal with statistics and probability. It seems that, until comparatively late in the school mathematics curriculum at the time, probability was covered exclusively by *public domain* discourse and, in particular, the concept ‘equally likely events’ was not mentioned at all. The omission of this fundamental concept is possibly the reason why so many of the students whom I taught when I was a secondary school mathematics teacher found probability to be an incomprehensible theme. There was lots of talk about dice and coins and spinners and I remember one question about a fox whose chance of catching its prey (hen, duck, …?) depended on the amount of cloud, which varied probabilistically! Utter nonsense, of course, but I’m still baffled by announcements by weather forecasts that put a probability on the likelihood of rain. Figure 3 shows the forecast for today in my current location. The probabilities are in the line of percentages between the wind speeds and the rain icons.

Figure 3. Kyoto weather 2nd – 3rd November 2022. (source BBC website[5])



So, at 17:00, it’s partially cloudy, 20° Celsius, with a northerly breeze of 5 Kph and a probability of rain at 2% or 0.02. What does this probability mean? Possibilities are, perhaps, that 2% of the area of Kyoto will experience rain, or it will rain 2% of the time, or that in 2% of situations when meteorological conditions are as they are now, there will be rain (how much, when, where, precisely)? The only clarity occurs after 23:00 tonight, when a probability of 0% indicates no chance of rain at all. I’ll make a point of writing to the meteorological office and asking them just what these probabilities mean, but you can see my confusion, I hope. Strictly speaking, this is *descriptive domain* text (from a mathematics point of view), but that doesn’t make it any clearer.

Returning to the headline about shortening your life by 30 minutes, as I’ve indicated, this cannot be a message to be taken literally by individuals: the online etymology dictionary gives the origin of the word ‘statistics’ as ‘science dealing with data

[4] The relevant chapter of this work was edited out of Dowling, 1998, for reasons of word length.

[5] https://www.bbc.com/weather/1857910

about the condition of a state or community' from 1770[6], which is consistent with Michel Foucault's (2007) discussion that identifies statistics with the management of populations, not individual life choices. So, is the headline ethical? My answer is that it is not, in fact, it's in effect a lie!

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I have been very critical of the school mathematics scheme *SMP 11-16* here and in previous publications. These criticisms were made from my position as a sociologist and educator. As someone having an interest in mathematics, however, my response to these textbooks, well, the higher track anyway, was far more appreciative. The attempt to base the discourse on set theory was consistent with 'modern mathematics' and some of the activities were highly imaginative. I found the initial version of the Advanced Level books, available just as I left university in 1972, to be quite exciting, starting off with a chapter on algebraic structure. I wondered why my head of department had handed the A-Level teaching over to me (my degree was in Physics). It became clear when these books were scrapped after only a few years and replaced with a new series with the more interesting stuff, including algebraic structure, abandoned or pushed into the Further Mathematics series. Apparently, few of the mathematics teachers at the time were familiar with this more advanced mathematics and many refused to teach it: the transmission model of pedagogy requires the teacher to be in control of the content as well as the students!

The *SMP 11-16* course is now rather old. The school books currently used are driven by State-led emphasis on examination performance and SATs rather than on the acquisition of mathematical knowledge *per se*. The current books, published by Pearson under the heading *Edexcel Mathematics*, weaken the grip of the *esoteric domain* as constructed in the earlier books by diluting the strictly mathematical *content*—perhaps the mathematical *expression* less so—and constituting an amalgam with another discourse. In Dowling, 2010, I referred to the second discourse as 'pedagogic theory', but the Edexcel books prioritise, not educational theory as such, but assessment in public examinations, which dominate the amalgam, so that school mathematics has become less a matter of "an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject", as the National Curriculum maintains, and more a training for passing GCSE examinations. Thus, for example, "The mathematics of trigonometry is ignored but rather [presented as] a set of procedures […] provided to answer exam questions" (Jeremy Burke, personal communication). The 'exam-style questions' that are included in the text even go so far as to provide a key to the number of marks to be awarded for each successfully completed part of the question, one example involving calculating the length of the 'opposite side', *x* cm, of a right-angled triangle ('NOT accurately drawn') is to be credited with '3 marks', comprising '1 mark for writing an equation using the correct ratio, 1 mark for rearranging and 1 mark for the correct value of *x*'. Since the hypotenuse is 32 cm and the relevant angle is 60º, I would simply state the answer as 16√3, which is precise, though in surd, not decimal form, so I would then have to render as approximation, 27.7 cm (using a remembered approximation for √3 (1.732)). I would presumably receive 1 mark for the answer, but miss out on the other 2 marks, even though my response showed evidence of a knowledge of Pythagoras' theorem and surds as

[6] https://www.etymonline.com/word/statistics

well as trigonometry and was a correct answer, which I was then required to approximate as a decimal. The marking of the 'exam-style' questions, here and throughout the *Edexcel* books seems detached from mathematical knowledge, reacting only to bureaucratically prescribed procedures.

So much for the *esoteric domain* in *Edexcel Mathematics*. The *public domain* retains the absurdity apparent in *SMP 11-16*: here are two examples about ratio.

> **Real** It takes 3 typists 5 hours to type a report
> How long would it take 7 typists
> Give your answer to the nearest minute

Well, are typists still employed to type reports? In what form are the draft reports handed to the typists? When they were so employed, were they paid by the hour or by the number of words/pages? In what sense is this a 'real' problem?

> 8 boys and 12 girls go to swimming lessons.
> In one lesson, the mean number of lengths swum by the boys is 4.5 and the mean number of lengths swum by the girls is 2.
> Work out the mean number of lengths swum by all the children in that lesson.

This item includes a 'hint':

> Hint Draw a bar to represent all the children.
> Split the bar into sections that show the numbers of boys and girls.
> Work out how many lengths the boys swam in total and how many lengths the girls swam in total. Work out how many lengths the children swam altogether.

The strategy suggested in the 'hint' seems to me to be rather more troublesome than solving the task mentally (the answer is 3). Is this 'problem' one that is likely to be encountered anywhere, by anyone? Here's a question from both the *Edexcel* 2018 GCSE Examination (first Foundation and first Higher papers) that is arguably even more absurd and that, in a sense, justifies the absurd nature of the textbook examples, the curriculum being exam-led:

> In a village
>
> > the number of houses and the number of flats are in the ratio 7:4
> > the number of flats and the number of bungalows are in the ratio 8:5
>
> There are 50 bungalows in the village
> How many houses are there in the village?

The numbers of houses, flats, and bungalows are already known, if the village is in existence and are, in any event, unlikely to be presented as ratios like this. Here, the answer, in case you were struggling (or insensible with laughing at the absurdity of the task), is 140.

I can't resist just one more, again from the Edexcel 2018 GCSE exam (this time from the second Foundation paper):

> Here is a list of ingredients for making 30 biscuits
>
> > Ingredients for 30 biscuits
> >
> > 225 g butter
> > 110 g caster sugar
> > 275 g plain flour
> > 75 g chocolate chips
>
> Lucas has the following ingredients.
>
> 900 g butter
> 1000 g caster sugar
> 1000 g plain flour
> 225 g chocolate chips
>
> What is the greatest number of biscuits Lucas can make?
> You must show your workings.

Again, not much working needed here: the chocolate chips are going to run out first, so 3 batches or 90 biscuits. But does cooking work like this? Well, not in my kitchen, where, for example, ingredient quantities are rarely, if ever, so sensitive, and as a Type 2 diabetes sufferer, I don't use sugar, but *Canderel*, which works out at 10 g per 100 g sugar, in terms of sweetness, so I'd need 33 g for 90 biscuits, or more, or less, if I wanted to vary the sweetness; the chocolate chips might pose a problem as well. I could go on, but all four of these *public domain* tasks are patently ridiculous, which I'm sure you can see for yourself (unless you're one of the authors of these books or examinations).

So, as with the *SMP 11-16* books, the public domain is composed of fictitious illustrations and in no way 'real' in the sense of presenting possible practical uses of mathematics, which claim is a lie! Is this ethical?

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Here's another challenge: is it ethical to condemn an educational programme without offering or at least hinting at a solution? If the answer is 'no', then I've been writing and speaking unethically for pretty much my entire academic career (not to mention my school teaching before that). Indeed, the possibility of a solution, of a route to an ethical education, is suggested in Figure 2 above and in the analyses of schooling that are driven by it. First, I have to announce that this relational space originally derived from a combination of an analysis of school mathematics texts that led to the generation of the 'theory' that is constituted in Figure 2 in an essentially Grounded Theory approach: the 'theory' was not imposed on the data but was built up from it. Of course, it also responded to my own 'theoretical sensitivity' (Glaser, 1978) and the comments of my doctoral supervisor, Basil Bernstein. My theoretical sensitivity included a familiarity with structural linguistics, which encouraged the analytic division of the linguistic sign into the categories, 'expression' and 'content' (Hjelmslev, 1970, and, originally, signifier/signified in Saussure, 1911) as well as, of course, the sociology of Basil Bernstein (for example, 1971), though I eventually reconceptualised his concept, 'classification' as strength of *institutionalisation* that does the work of both 'classification' and 'framing' (see Dowling, 1998). My 'theory', which I refer to as a 'method', including the relational space in Figure 2 has been productively deployed in a wide range of settings. If it is reasonable to suppose that every activity that is recognisable as distinct, is so by virtue of its practitioners having constructed an *esoteric domain*, then the scheme in Figure 2 plausibly describes all stable activities.

So, the problems with mathematics education that I have revealed here and elsewhere are related to the prevalence of its *esoteric domain* or the hybrid domains that are constituted by an amalgam with pedagogic or assessment theory. In each case, the operational *esoteric domain*, in its singular or hybrid forms, regulates the practice. In Dowling, 2010, I made a distinction between *fetch* and *push* strategies operated by the *gaze* of the subject of the *esoteric domain*—in the case of mathematics education, a teacher or the author of a textbook or examination question. A *fetch* strategy might be thought of as collecting material from another activity and reorganising it according to the principles of the fetching *esoteric domain*. A *push* strategy projects the result of such reorganising back into the other activity. Whilst this might be described as 'mathematical modelling', in the case of a mathematics *esoteric domain*, it is, as I have illustrated, a distortion, a recontextualisation of the original source activity and this is why the *public domain* illustrations in this article look so comical, pretending to be what they patently are not! Comedy is not an necessary feature of recontextualization, but distortion is. I am suggesting that this scheme is not limited to mathematics education, but might be applied to any stable activity, independently of the culture that it is addressing. This is a plausible suggestion because the dimensions, 'expression' and 'content' and the concepts of the relational space are defined at a high level of etic abstraction and not tied to any emic categories.

I am not asserting that this is the way things really are, because I am not working in a naïvely realist epistemology, but simply offering an invitation: suppose you look at it like this, what follows?

Well then, if the *esoteric domain* of any activity structures the analysis produced by the *fetch* and *push* actions of the *gaze* of its subject, then this domain must be located in the activity that generates the original problem to be solved. Social problems—police action structured along race lines, social and health problems associated with alcohol consumption, Educational success distributed ethnically—are often revealed in their extent by quantitative research and statistical analysis. This is recognised, but this kind of research on its own cannot tell you what's going on, what is causing the statistics to come out as they do and what individual variations do they conceal. What is needed is a shift away from the prevalence of quantification, from mathematical domination. The three social problems that I've listed here will benefit from qualitative research that gets close to the action where the problems may become visible at a micro level. It is at this level they must be addressed and potentially solved. The move to counting should not be initiated until we know exactly what it is that we want to count and the complexity of our structure of variables should be understood qualitatively before we start to enumerate, because counting effaces the respective individualities of the items that are counted.

Furthermore, the terrain—the chronotope—over which the problem in focus extends will consist of diverse contexts and the precise nature of the problem will vary accordingly. The conditions that are responsible will also vary with the context and so solutions will, of necessity, be local as well. It may be that a solution in one context will suggest solutions in contexts that are similar in some way, but these solutions will need to be explored and, if necessary tested, also at the micro level.

The problem of the racially structured profile of examination outcomes that was presented to me by the staff at my South African host university suggested that quantification had taken place. My response, what in their practice might be responsible for this? would seem to require a qualitative analysis of the practice, perhaps an ethnographically-inspired study, not only to get in close to this practice, but because students are not exclusively defined as African South Africans, indeed, one might imagine that the range of variables that this descriptor effaces is extensive indeed; the human genome, for example, consists of 2.0-2.5.10^4 genes[7], not to mention individual and group cultural variations. In general, though, any analysis will have been produced by a methodological technology, we should be asking, what is it about this technology that has produced the results that we are presented with. Because we are presented with a *public domain* narrative that has been *pushed* from an *esoteric domain* that is alienated from the context of our concern—social injustice, perhaps, not mathematics, as such. The narrative is not only a distortion of this context, but the *esoteric domain* itself may be suffering from a 'bug'.

This conclusion reinstates the importance of the study and development of mathematics—the discipline under consideration—and also insists on the dismantling of *public domain* recontextualisations that misunderstand the settings into which they are *pushed*. If you want to study/teach mathematics, then do that, because it's a worthwhile area of study. If you want to address social injustice, then start there and recruit from the disciplines where appropriate. This is precisely the strategy of Annemarie Mol and John Law in their 'topological' study of anaemia:

> We have taken the notion of 'topology' from mathematics, and, in the process of bringing it to social theory, we have necessarily also altered it.
>
> Mol & Law, 1994; 643

Mol and Law's study deploys a mathematical concept—topology—in describing space in different ways, as regions, networks and as flows, but their research is qualitative, based on interview and observation data. The focus of their study—anaemia—transforms as we move between topologies: their ontology is fluid as is their 'flow' topological space and as, indeed, is the mathematics that they recontextualise.

In an article that is referenced as 'forthcoming' in Mol &. Law's paper, but, of course, is by now available, Mol, this time writing with Marc Berg, reports on a study involving an analysis of medical textbooks and observation and interviews with medical practitioners in the Netherlands, they conclude that:

> Using personal hemoglobin [sic] standards, and working with standards based on population statistics, gives the same results in many but not all patients. Some patients will be diagnosed as anemic according to one logic and not according to the other. And the clinical detection of anemics implies a different organization of health care than that suggested by the need to find all statistical deviants.
>
> Mol & Berg, 1994; 259

The use of statistics that is referred to here defines the critical concentration of Hb as two standard deviations below the mean. If the distribution is normal (which it is presumed to be) then 95% of the population will have Hb levels above this. Mol and Berg's article, however, argues that this is only one of a number of logics used to determine anaemia. There are different logics and so different anaemias. The potential for confusion here is annulled by the distinction, commonly made in medicine, between 'pure' principles and practice, which can reasonably be expected to be contaminated by technical errors and social interests etc. Even

[7] https://medlineplus.gov/genetics/understanding/basics/gene/

sticking with the 'pure', however, the cut-off point is a statistical arbitrary, an artefact of *esoteric domain* mathematics, which recontextualises the lives of individuals who may or may not be suffering from anaemia. Again, Mol and Berg's qualitative study reveals not only the mathematical distortion, but also the hybrid nature of medical knowledge that sustains the co-existence of multiple logics.

Social problems, and the global variation in the incidence and in the construction of anaemia is certainly one of these as Mol and Law's article illustrates, are locally visible in the experience, in the narratives of the individuals who suffer them and in the actions of the individuals who are responsible for them as well as those who are attempting to remediate them. If this is what we want to address, then we must begin with these experiences and actions; tricky, eh? Much easier to start with a nice, simple survey and SPSS and certainly much easier to put in a textbook or an examination question, but is this ethical?

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